hep-ph/0103xxx

Effective Lagrangian for $\overline{s}bg$ and $\overline{s}b\gamma$ Vertices in the mSUGRA model

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Abstract

Complete expressions of the $\overline{s}bg$ and $\overline{s}b\gamma$ vertices are derived in the framework of supersymmetry with minimal flavor violation. With the minimal supergravity (mSUGRA) model, a numerical analysis of the supersymmetric contributions to the Wilson Coefficients at the weak scale is presented.

PACS numbers: 11.30.Er, 12.15.Ff, 12.60.Jv, 13.10+q.

Keywords: Supersymmetry, Flavor violation, Effective Lagrangian.

1 Introduction

The rare B decays serve as a good test for new physics beyond the standard model (SM) since they are not seriously affected by the uncertainties due to long distance effects. The forthcoming B-factories will make more precise measurements on the rare B-decay processes and those measurements would set more strict constraints on the new physics beyond SM. The main purpose of investigation of B-decays, especially the rare decay modes is to search for traces of new physics and determine its parameter space. In all the extensions of SM, the supersymmetry is considered as one of the most plausible candidates. In the general supersymmetric extension of SM, new sources of flavor violation may appear in those soft breaking terms[1]. Applying the mass insertion method, the influence of those non-universal soft breaking terms on various flavor changing neutral current (FCNC) processes are discussed in literatures[2]. However, too many free parameters which exist in the supersymmetry model with non-universal soft breaking terms decrease the model prediction ability. Thus for a practical calculation whose results can be compared with the data, one needs to reduce the number

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of the free parameters in some way, i.e. by enforcing some physical conditions and assuming reasonable symmetries. A realization of this idea is the minimal supergravity (mSUGRA), which is fully specified by only five parameters[3]. In this work, we perform a strict analysis on the $\bar{s}bg$ ($\bar{s}b\gamma$) effective Lagrangian in the minimal flavor violation supersymmetry up to the leading order. The NLO SUSY-QCD corrections to those processes have been evaluated in our another work [4].

The most general form of the superpotential which does not violate gauge invariance and the conservation laws in SM is

$$\mathcal{W} = \mu\epsilon_{ij}\hat{H}_i^1\hat{H}_j^2 + \epsilon_{ij}h_l^I\hat{H}_i^1\hat{L}_j^I\hat{R}^I - h_d^I(\hat{H}_1^1\hat{Q}_2^I - \hat{H}_2^1 V^{IJ}\hat{Q}_1^J)\hat{D}^I - h_u^I(\hat{H}_1^2 V^{*JI}\hat{Q}_2^J - \hat{H}_2^2\hat{Q}_1^I)\hat{U}^I. \quad (1)$$

Here $\hat{H}^1$, $\hat{H}^2$ are Higgs superfields; $\hat{Q}^I$ and $\hat{L}^I$ are quark and lepton superfields in doublets of the weak SU(2) group, where I=1, 2, 3 are the indices of generations; the rest superfields $\hat{U}^I$, $\hat{D}^I$ and $\hat{R}^I$ are quark superfields of the u- and d-types and charged leptons in singlets of the weak SU(2) respectively. Indices i, j are contracted for the SU(2) group, and h_l, $h_{u,d}$ are the Yukawa couplings. In order to break the supersymmetry, the soft breaking terms are introduced as

$$\begin{aligned}\mathcal{L}_{soft} = &-m_{H^1}^2 H_i^{1*}H_i^1 - m_{H^2}^2 H_i^{2*}H_i^2 - m_{L^I}^2\tilde{L}_i^{I*}\tilde{L}_i^I \\ &-m_{R^I}^2\tilde{R}^{I*}\tilde{R}^I - m_{Q^I}^2\tilde{Q}_i^{I*}\tilde{Q}_i^I - m_{U^I}^2\tilde{U}^{I*}\tilde{U}^I \\ &-m_{D^I}^2\tilde{D}^{I*}\tilde{D}^I + (m_1\lambda_B\lambda_1 + m_2\lambda_A^i\lambda_A^i \\ &+m_3\lambda_G^a\lambda_G^a + h.c.) + \Big[B\mu\epsilon_{ij}H_i^1H_j^2 + \epsilon_{ij}A_l^I h_l^I H_i^1\tilde{L}_j^I\tilde{R}^I \\ &-A_d^I h_d^I(H_1^1\tilde{Q}_2^I - H_2^1 V^{IJ}\tilde{Q}_1^J)\tilde{D}^I - A_u^I h_u^I(H_1^2 V^{*JI}\tilde{Q}_2^J - H_2^2\tilde{Q}_1^I)\tilde{U}^I + h.c.\Big],\end{aligned} \quad (2)$$

where $m_{H^1}^2$, $m_{H^2}^2$, $m_{L^I}^2$, $m_{R^I}^2$, $m_{Q^I}^2$, $m_{U^I}^2$ and $m_{D^I}^2$ are the parameters in unit of mass squared, m_3, m_2, m_1 denote the masses of λ_G^a ($a = 1,\ 2,\ \cdots\ 8$), λ_A^i ($i = 1,\ 2,\ 3$) and λ_B, which are the $SU(3)\times SU(2)\times U(1)$ gauginos. B is a free parameter in unit of mass. A_l^I, A_u^I, A_d^I ($I = 1,\ 2,\ 3$) are the soft breaking parameters that result in mass splitting between leptons, quarks and their supersymmetric partners. Taking into account of the soft breaking terms Eq.(2), we can study the phenomenology within the minimal supersymmetric extension of the standard model (MSSM). The resultant mass matrix of the up-type scalar quarks is written as

$$m_{\tilde{U}^I}^2 = \begin{pmatrix} m_{Q^I}^2 + m_{u^I}^2 + (\frac{1}{2} - \frac{2}{3}\sin^2\theta_{\rm W})\cos 2\beta m_{\rm Z}^2 & -m_{u^I}(A_u^I + \mu\cot\beta) \\ -m_{u^I}(A_u^I + \mu\cot\beta) & m_{U^I}^2 + m_{u^I}^2 + \frac{2}{3}\sin^2\theta_{\rm W}\cos 2\beta m_{\rm Z}^2 \end{pmatrix}, \quad (3)$$

and the corresponding mass matrix of the down-type scalar quarks is

$$m_{\tilde{D}^I}^2 = \begin{pmatrix} m_{Q^I}^2 + m_{d^I}^2 + (\frac{1}{2} + \frac{1}{3}\sin^2\theta_{\rm W})\cos 2\beta m_{\rm Z}^2 & -m_{d^I}(A_d^I + \mu\tan\beta) \\ -m_{d^I}(A_d^I + \mu\tan\beta) & m_{D^I}^2 + m_{d^I}^2 - \frac{1}{3}\sin^2\theta_{\rm W}\cos 2\beta m_{\rm Z}^2 \end{pmatrix}, \quad (4)$$

with m_{u^I}, m_{d^I} ($I = 1,\ 2,\ 3$) being the masses of the I-th generation quarks. One difference between the MSSM and SM is the Higgs sector. There are four charged scalars, two of them are physical massive Higgs bosons and other are massless Goldstones in the SUSY extension. The mixing matrix can be written as:

$$\mathcal{Z}_H = \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \quad (5)$$

with $\tan\beta = \frac{\upsilon_2}{\upsilon_1}$ and υ_1, υ_2 being the vacuum-expectation values of the two Higgs scalars. Another matrix that we will use in the later derivation is the chargino mixing matrix. The SUSY partners of the charged Higgs and $W^\pm$ combine to give four Dirac fermions: $\chi_1^\pm$, $\chi_2^\pm$. The two mixing matrices $\mathcal{Z}^\pm$ appearing in the Lagrangian are defined as

$$(\mathcal{Z}^-)^T\mathcal{M}_c\mathcal{Z}^+ = diag(m_{\chi_1}, m_{\chi_2}), \quad (6)$$

where $\mathcal{M}_c$ is the mass matrix of charginos. In a similar way, $Z_{U,D}$ diagonalize the mass matrices of the up- and down-type squarks respectively:

$$\mathcal{Z}^{\dagger}_{\tilde{U}^I} m^2_{\tilde{U}^I} \mathcal{Z}_{\tilde{U}^I} = diag(m^2_{\tilde{U}^I_1},\ m^2_{\tilde{U}^I_2}) ,$$
$$\mathcal{Z}^{\dagger}_{\tilde{D}^I} m^2_{\tilde{D}^I} \mathcal{Z}_{\tilde{D}^I} = diag(m^2_{\tilde{D}^I_1},\ m^2_{\tilde{D}^I_2}) . \tag{7}$$

In the framework of minimal supergravity (mSUGRA), the unification assumptions at the GUT scale are expressed as[5]

$$A^I_l = A^I_d = A^I_u = A_0 , \tag{8}$$
$$B = A_0 - 1 ,$$
$$m^2_{H^1} = m^2_{H^2} = m^2_{L^I} = m^2_{R^I} = m^2_{Q^I} = m^2_{U^I} = m^2_{D^I} = m^2_0 ,$$
$$m_1 = m_2 = m_3 = m_{\frac{1}{2}} .$$

Under these assumptions, the mSUGRA is specified by five parameters:

$$A_0,\ m_0,\ m_{\frac{1}{2}},\ \tan\beta,\ sgn(\mu),$$

and the flavor structure of the model is similar to SM, i.e. flavors change only via the CKM matrix.

The supersymmetric contributions will modify the Wilson Coefficients of the effective $\overline{s}bg$ and $\overline{s}b\gamma$ vertices. For the W-boson propagator, we adopt the nonlinear R_ξ gauge whose gauge fixing term is [6]

$$\mathcal{L}_{gauge-fixing} = -\frac{1}{\xi} f^{\dagger} f \tag{9}$$

with $f = (\partial_\mu W^{+\mu} - ieA_\mu W^{+\mu} - i\xi m_{\rm W}\phi^+)$ in our calculations. A thorough discussion about the gauge invariance was given by Deshpande et al. [7, 8].

As in the case of SM [13], the operator basis for $b \to sg$ in the supersymmetry consists of

$$\mathcal{O}_1 = \frac{1}{(4\pi)^2}\bar{s}(i\not{D})^3\omega_- b ,$$
$$\mathcal{O}_2 = \frac{1}{(4\pi)^2}\bar{s}\{i\not{D}, g_s G\cdot\sigma\}\omega_- b ,$$
$$\mathcal{O}_3 = \frac{1}{(4\pi)^2}\bar{s}iD_\mu(ig_s G^{\mu\nu})\gamma_\nu\omega_- b ,$$
$$\mathcal{O}_4 = \frac{1}{(4\pi)^2}\bar{s}(i\not{D})^2(m_s\omega_- + m_b\omega_+)b ,$$
$$\mathcal{O}_5 = \frac{1}{(4\pi)^2}\bar{s}g_s G\cdot\sigma(m_s\omega_- + m_b\omega_+)b . \tag{10}$$

In these operators, $D_\mu \equiv \partial_\mu - ig_s G_\mu$ and $G_{\mu\nu} \equiv G^a_{\mu\nu}T^a$ denotes the gluon field strength tensor with $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc}G^b_\mu G^c_\nu$, and $G\cdot\sigma \equiv G_{\mu\nu}\sigma^{\mu\nu}$.

For transition $b \to s\gamma$, the operator basis is somewhat different from those in eq.(10) and the changes are reflected in the following replacements:

$$\mathcal{O}_2 \to \mathcal{O}_6 = \frac{1}{(4\pi)^2}\bar{s}\{i\not{D}, eQ_d F\cdot\sigma\}\omega_- b ,$$
$$\mathcal{O}_3 \to \mathcal{O}_7 = \frac{1}{(4\pi)^2}\bar{s}iD_\mu(ieQ_d F^{\mu\nu})\gamma_\nu\omega_- b ,$$
$$\mathcal{O}_5 \to \mathcal{O}_8 = \frac{1}{(4\pi)^2}\bar{s}eQ_d F\cdot\sigma(m_s\omega_- + m_b\omega_+)b \tag{11}$$

with $F_{\mu\nu}$ being the electromagnetic field strength tensor and $F\cdot\sigma \equiv F_{\mu\nu}\sigma^{\mu\nu}$.

2 The effective Lagrangian for $\bar{s}bg$ ($\bar{s}b\gamma$)

At first, we present the analysis of $\bar{s}b$-mixing. The self-energy diagrams are drawn in Fig.1. The unrenormalized $\bar{s}b$ self-energy is given as

$$\Sigma = \frac{ig_2^2}{32\pi^2}\sum_{i=u,c,t} V_{ib}V_{is}^*\Big\{\Big(A_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) + \frac{p^2}{m_{\rm W}^2}A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\Big)\not{p}\omega_-$$
$$+\Big(B_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) + \frac{p^2}{m_{\rm W}^2}B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\Big)(m_s\omega_- + m_b\omega_+)$$
$$+C_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\frac{m_bm_s}{m_{\rm W}^2}\not{p}\omega_+\Big\} \tag{12}$$

with the symbolic definitions $x_i = \frac{m_i^2}{m_{\rm W}^2}, x_H = \frac{m_{H^+}^2}{m_{\rm W}^2}, x_{\tilde{U}^i_\alpha} = \frac{m_{\tilde{U}^i_\alpha}^2}{m_{\rm W}^2}, x_{\chi_\beta} = \frac{m_{\chi_\beta}^2}{m_{\rm W}^2}$ with $i = u\ ,c\ ,t$. Those form factors A_0, A_1, B_0, B_1 and C_0 are complicated functions of the parameters and their explicit expressions are collected in Appendix.A.

We renormalize the $\bar{s}b$ self-energy according to the well-known prescription, namely by demanding that the renormalized self-energy $\hat{\Sigma}$ vanishes when one of the external legs is on its mass-shell[9, 10, 11]. Obviously, this is a necessary physical condition which must be satisfied. This is realized as

$$\hat{\Sigma} = \frac{ig_2^2}{32\pi^2}\sum_{i=u,c,t} V_{ib}V_{is}^*\Big\{\Big(A^* + A_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) + \frac{p^2}{m_{\rm W}^2}A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\Big)\not{p}\omega_-$$
$$+\Big(B_s^* + B_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) + \frac{p^2}{m_{\rm W}^2}B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\Big)m_s\omega_-$$
$$+\Big(B_b^* + B_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) + \frac{p^2}{m_{\rm W}^2}B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\Big)m_b\omega_+$$
$$+\Big(C^* + C_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\Big)\frac{m_bm_s}{m_{\rm W}^2}\not{p}\omega_+\Big\}\ , \tag{13}$$

where

$$A^* = -A_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) - \frac{m_b^2+m_s^2}{m_{\rm W}^2}\Big(A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) + B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\Big)\ ,$$
$$B_b^* = -B_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) - \frac{m_s^2}{m_{\rm W}^2}\Big(A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) + B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\Big)\ ,$$
$$B_s^* = -B_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) - \frac{m_b^2}{m_{\rm W}^2}\Big(A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) + B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\Big)\ ,$$
$$C^* = -\frac{m_bm_s}{m_{\rm W}^2}\Big(A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) + 2B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) + C_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\Big)\ . \tag{14}$$

After carrying out the renormalization procedure described above, the self-energy is written as

$$\hat{\Sigma} = \frac{ig_2^2}{32\pi^2}\sum_{i=u,c,t} V_{ib}V_{is}^*\Big\{\Big[\frac{p^2-m_b^2-m_s^2}{m_{\rm W}^2}A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) - \frac{m_b^2+m_s^2}{m_{\rm W}^2}B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\Big]\not{p}\omega_-$$

$$+\Big[\frac{p^2}{m_{\rm W}^2}B_1(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})+\frac{m_b^2}{m_{\rm W}^2}\Big(A_1(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})+B_1(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})\Big)\Big]m_s\omega_-$$
$$+\Big[\frac{p^2}{m_{\rm W}^2}B_1(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})+\frac{m_s^2}{m_{\rm W}^2}\Big(A_1(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})+B_1(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})\Big)\Big]m_b\omega_+$$
$$-\frac{m_bm_s}{m_{\rm W}^2}\Big(A_1(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})+2B_1(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})\Big)\not p\omega_+\Big\}. \tag{15}$$

This procedure is exactly the same as that adopted in the SM case[18].

Next, let us calculate the unrenormalized $\bar sbg$ vertex $\Gamma_\rho(p,q)$ corresponding to Fig.2. Keeping terms up to order $\frac{p^2,\,q^2}{m_{\rm W}^2}$[12, 13, 15, 16, 17], we have

$$\Gamma_\rho^{b\to sg}=g_sT^a\frac{ig_2^2}{32\pi^2}\sum_{i=u,c,t}V_{ib}V_{is}^*\Big\{A_0(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})\gamma_\rho\omega_-$$
$$+A_1(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})\frac{p^2\gamma_\rho+(p+q)^2\gamma_\rho+\not p\gamma_\rho\not p}{m_{\rm W}^2}\omega_-+F_1(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})\frac{q^2}{m_{\rm W}^2}\gamma_\rho\omega_-$$
$$+F_2(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})\frac{\not p\gamma_\rho\not q}{m_{\rm W}^2}\omega_-+F_3(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})\frac{\not q\gamma_\rho\not p}{m_{\rm W}^2}\omega_-$$
$$+F_4(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})\frac{\not q\gamma_\rho\not q}{m_{\rm W}^2}\omega_-+B_1(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})\frac{1}{m_{\rm W}^2}\Big((\not p+\not q)\gamma_\rho+\gamma_\rho\not p\Big)(m_s\omega_-+m_b\omega_+)$$
$$+F_5(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})\frac{1}{m_{\rm W}^2}[\not q,\gamma_\rho](m_s\omega_-+m_b\omega_+)+C_0(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})\frac{m_bm_s}{m_{\rm W}^2}\gamma_\rho\omega_+\Big\}\;, \tag{16}$$

where $F_i(x_i,x_{_H},x_{_{\tilde U^i_\alpha}},x_{\chi_\beta})$ $(i=1,\ \cdots,\ 5)$ are collected in Appendix.A. From Eq.12 and Eq.16, it is easy to show that $\Gamma_\rho^{b\to sg}$ obeys the Ward-Takahashi identity

$$q^\rho\Gamma_\rho^{b\to sg}(p,q)=g_sT^a\Big[\Sigma(p+q)-\Sigma(p)\Big]\;. \tag{17}$$

According to the general principle of renormalization, $\bar sbg-$vertex does not exist in the fundamental Lagrangian, thus it does not need to be renormalized. In other words, the divergence would be canceled as the physical conditions are taken into account. In the nonlinear R_ξ gauge, as well as in the unitary gauge, the one-loop penguin diagram results in a divergence. On other side, all the one-loop diagrams which contribute to the $b\to sg$ or $b\to s\gamma$ processes constitute a convergent subgroup. Thus obviously, the renormalizations of the penguin and flavor-changing self energies are associated. In fact, the Ward-Takahashi identity holds at the unrenormalized penguin, to renormalize the $\bar sbg$ vertex, we demand that the Ward-Takahashi identity be preserved by the renormalized vertex $\hat\Gamma_\rho^{b\to sg}$[18],

$$q^\rho\hat\Gamma_\rho^{b\to sg}(p,q)=g_sT^a\Big[\hat\Sigma(p+q)-\hat\Sigma(p)\Big]\;. \tag{18}$$

It is noted that with this requirement, just as in the SM case [18], the renormalization of the $\bar sbg$ vertex is realized when we renormalize the self-energy by enforcing the physical condition $\hat\Sigma=0$ as one of the external legs being on its mass shell. Moreover, indeed, the renormalization scheme of the $\bar sbg$ vertex pledges the current conservation for an on-shell transition, since the renormalized self-energies $\hat\Sigma(p+q)$ and $\hat\Sigma(p)$ are zero as both b and s are on mass shell [18].

This renormalization scheme can be understood from another angle. The requirement that the Ward-Takahashi identity holds and condition $\hat\Sigma$(on-shell)=0 realize the renormalization of the $\bar sbg$ vertex and the scheme is equivalent to summing up the contributions of penguin and flavor-changing self-energies to the transition $\bar sbg$ at one-loop level. This procedure can be generalized to two-loop calculations.

Applying Eq.18, we have

$$\hat{\Gamma}_{\rho}^{b\to sg} = g_s T^a \frac{ig_2^2}{32\pi^2}\sum_{i=u,c,t} V_{ib}V_{is}^*\Big\{-\frac{m_b^2+m_s^2}{m_{\rm W}^2}\Big(A_1(x_i,x_{_H},x_{\tilde{U}_\alpha^i},x_{\chi_\beta})+B_1(x_i,x_{_H},x_{\tilde{U}_\alpha^i},x_{\chi_\beta})\Big)\gamma_\rho\omega_-$$

$$+A_1(x_i,x_{_H},x_{\tilde{U}_\alpha^i},x_{\chi_\beta})\frac{p^2\gamma_\rho+(p+q)^2\gamma_\rho+\not p\gamma_\rho\not p}{m_{\rm W}^2}\omega_- + F_1(x_i,x_{_H},x_{\tilde{U}_\alpha^i},x_{\chi_\beta})\frac{q^2}{m_{\rm W}^2}\gamma_\rho\omega_-$$

$$+F_2(x_i,x_{_H},x_{\tilde{U}_\alpha^i},x_{\chi_\beta})\frac{\not p\gamma_\rho\not q}{m_{\rm W}^2}\omega_- + F_3(x_i,x_{_H},x_{\tilde{U}_\alpha^i},x_{\chi_\beta})\frac{\not q\gamma_\rho\not p}{m_{\rm W}^2}\omega_-$$

$$+F_4(x_i,x_{_H},x_{\tilde{U}_\alpha^i},x_{\chi_\beta})\frac{\not q\gamma_\rho\not q}{m_{\rm W}^2}\omega_- + B_1(x_i,x_{_H},x_{\tilde{U}_\alpha^i},x_{\chi_\beta})\frac{1}{m_{\rm W}^2}\Big((\not p+\not q)\gamma_\rho+\gamma_\rho\not p\Big)(m_s\omega_-+m_b\omega_+)$$

$$+F_5(x_i,x_{_H},x_{\tilde{U}_\alpha^i},x_{\chi_\beta})\frac{1}{m_{\rm W}^2}[\not q,\gamma_\rho](m_s\omega_-+m_b\omega_+)$$

$$-\Big(A_1(x_i,x_{_H},x_{\tilde{U}_\alpha^i},x_{\chi_\beta})+2B_1(x_i,x_{_H},x_{\tilde{U}_\alpha^i},x_{\chi_\beta})\Big)\frac{m_bm_s}{m_{\rm W}^2}\gamma_\rho\omega_+\Big\}. \qquad (19)$$

The terms of dimension-four which are related to the $\bar{s}\gamma_\rho\omega_\pm b$ vertex cancel each other as long as we let b and s quarks be on their mass shells[13], so that we do not need to consider them at all. We ignore all terms which vanish as $\frac{m_{u,c}^2}{m_{\rm W}^2}\to 0$, whereas keep the part in the coefficients which are proportional to $\ln\frac{m_{u,c}^2}{m_{\rm W}^2}$ in the final effective vertex for $b\to sg$, we can recast Eq.19 to a form with the operator basis given in Eq.10:

$$\hat{\Gamma}_{\rho}^{b\to sg} = \frac{4G_F}{\sqrt{2}}\Big\{V_{tb}V_{ts}^*\sum_{i=1}^5 C_i(\mu_{\rm W})\mathcal{O}_i+\Big(\frac{4}{3}V_{cb}V_{cs}^*\ln x_c+\frac{4}{3}V_{ub}V_{us}^*\ln x_u\Big)\mathcal{O}_3\Big\}\ . \qquad (20)$$

After matching between the effective theory and the full theory[14], we have the effective Lagrangian for $b\to sg$ at the weak scale in the minimal flavor violating supersymmetry as:

$$\mathcal{L}_{b\to sg} = \frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\sum_{i=1}^5 C_i(\mu_{\rm W})\mathcal{O}_i \qquad (21)$$

where

$$C_1(\mu_{\rm W}) = -\Big[\frac{5x_t+1}{2(1-x_t)^3}+\frac{x_t^3+2x_t^2}{(1-x_t)^4}\ln x_t\Big]+\frac{1}{\tan^2\beta}\Big[\frac{x_t^3x_{_H}(\ln x_t-\ln x_{_H})}{(x_{_H}-x_t)^4}$$

$$+\frac{2x_t^3+5x_t^2x_{_H}-x_tx_{_H}^2}{6(x_{_H}-x_t)^3}\Big]+2\sum_{\alpha,\,\beta}(\mathcal{A}_3^{\alpha,\beta})^2\Big[-\frac{x_{\chi_\beta}^2x_{\tilde{U}_\alpha^3}(\ln x_{\tilde{U}_\alpha^3}-\ln x_{\chi_\beta})}{(x_{\tilde{U}_\alpha^3}-x_{\chi_\beta})^4}$$

$$-\frac{2x_{\chi_\beta}^2+5x_{\chi_\beta}x_{\tilde{U}_\alpha^3}-x_{\tilde{U}_\alpha^3}^2}{6(x_{\chi_\beta}-x_{\tilde{U}_\alpha^3})^3}\Big]\ ,$$

$$C_2(\mu_{\rm W}) = x_t\Big[\frac{5x_t-2}{4(1-x_t)^4}\ln x_t+\frac{-4+13x_t-3x_t^2}{8(1-x_t)^3}\Big]+\frac{1}{\tan^2\beta}\Big[-\frac{x_t^2x_{_H}^2(\ln x_t-\ln x_{_H})}{4(x_{_H}-x_t)^4}$$

$$-\frac{2x_tx_{_H}^2+5x_t^2x_{_H}-x_t^3}{24(x_{_H}-x_t)^3}\Big]+\sum_{\alpha,\,\beta}(\mathcal{A}_3^{\alpha,\beta})^2\Big[\frac{x_{\chi_\beta}^2x_{\tilde{U}_\alpha^3}(\ln x_{\tilde{U}_\alpha^3}-\ln x_{\chi_\beta})}{2(x_{\chi_\beta}-x_{\tilde{U}_\alpha^3})^4}$$

$$+\frac{2x_{\chi_\beta}^2+5x_{\chi_\beta}x_{\tilde{U}_\alpha^3}-x_{\tilde{U}_\alpha^3}^2}{12(x_{\chi_\beta}-x_{\tilde{U}_\alpha^3})^3}\Big]\ ,$$

$$
\begin{aligned}
C_3(\mu_{\rm W}) =& \Big[\frac{-9x_t^2+16x_t-4}{6(1-x_t)^4}\ln x_t+\frac{-x_t^3-11x_t^2+18x_t}{12(1-x_t)^3}\Big] \\
&+\frac{1}{\tan^2\beta}\Big[\frac{(2x_tx_{_H}^3-3x_t^2x_{_H}^2)(\ln x_t-\ln x_{_H})}{6(x_{_H}-x_t)^4}+\frac{16x_tx_{_H}^2-29x_t^2x_{_H}+7x_t^3}{36(x_{_H}-x_t)^3}\Big] \\
&+\sum_{\alpha,\,\beta}(\mathcal{A}_3^{\alpha,\beta})^2\Big[\frac{x_{\chi_\beta}^3(\ln x_{\tilde U_\alpha^3}-\ln x_{\chi_\beta})}{3(x_{\chi_\beta}-x_{\tilde U_\alpha^3})^4}+\frac{11x_{\chi_\beta}^2-7x_{\chi_\beta}x_{\tilde U_\alpha^3}+2x_{\tilde U_\alpha^3}^2}{18(x_{\chi_\beta}-x_{\tilde U_\alpha^3})^3}\Big]\;, \\
C_4(\mu_{\rm W}) =& x_t\Big[\frac{x_t^2-x_t}{(1-x_t)^4}\ln x_t+\frac{x_t^2-1}{2(1-x_t)^3}\Big]-\Big[\frac{x_t^2x_{_H}(\ln x_t-\ln x_{_H})}{(x_{_H}-x_t)^3}+\frac{x_{_H}x_t+x_t^2}{2(x_{_H}-x_t)^2}\Big] \\
&-\sum_{\alpha,\,\beta}\frac{m_{\chi_\beta}}{\sqrt2 m_{\rm W}\cos\beta}(\mathcal{A}_3^{\alpha,\beta}\mathcal{B}_3^{\alpha,\beta})\Big[\frac{2x_{\chi_\beta}x_{\tilde U_\alpha^3}(\ln x_{\tilde U_\alpha^3}-\ln x_{\chi_\beta})}{(x_{\chi_\beta}-x_{\tilde U_\alpha^3})^3}+\frac{x_{\chi_\beta}+x_{\tilde U_\alpha^3}}{(x_{\chi_\beta}-x_{\tilde U_\alpha^3})^2}\Big]\;, \\
C_5(\mu_{\rm W}) =& x_t\Big[\frac{\ln x_t}{2(1-x_t)^3}+\frac{3-x_t}{4(1-x_t)^2}\Big]+\Big[\frac{x_tx_{_H}^2(\ln x_t-\ln x_{_H})}{2(x_{_H}-x_t)^3}+\frac{3x_{_H}x_t-x_t^2}{4(x_{_H}-x_t)^2}\Big] \\
&-\sum_{\alpha,\,\beta}\frac{m_{\chi_\beta}}{\sqrt2 m_{\rm W}\cos\beta}(\mathcal{A}_3^{\alpha,\beta}\mathcal{B}_3^{\alpha,\beta})\Big[\frac{x_{\chi_\beta}x_{\tilde U_\alpha^3}(\ln x_{\tilde U_\alpha^3}-\ln x_{\chi_\beta})}{(x_{\chi_\beta}-x_{\tilde U_\alpha^3})^3}+\frac{x_{\chi_\beta}+x_{\tilde U_\alpha^3}}{2(x_{\chi_\beta}-x_{\tilde U_\alpha^3})^2}\Big]\;,
\end{aligned} \tag{22}
$$

with

$$
\begin{aligned}
\mathcal{A}_i^{\alpha,\beta} &= -\mathcal{Z}_{\tilde U^i}^{1,\alpha}\mathcal{Z}_{1,\beta}^{+}+\frac{m_{u_i}}{\sqrt2 m_{\rm W}\sin\beta}\mathcal{Z}_{\tilde U^i}^{2,\alpha}\mathcal{Z}_{2,\beta}^{+}\;, \\
\mathcal{B}_i^{\alpha,\beta} &= -\mathcal{Z}_{\tilde U^i}^{1,\alpha}\mathcal{Z}_{2,\beta}^{-}\;,
\end{aligned} \tag{23}
$$

and the mixing matrices $\mathcal{Z}_{\tilde U^i}\mathcal{Z}_{2,\beta}^{\pm}$ are given in Eqs.6, 7. The first terms in the above expressions are the SM contributions[13] and the second terms are the charged Higgs contributions. The supersymmetric corrections exist in the third terms.

For the vertex $\bar s b\gamma$, the Feynman diagrams are drawn in Fig.3.

With all unrenormalized quantities the Ward-Takahashi identity for the $\bar s b\gamma$ vertex is in form:

$$
q^\rho\Gamma_\rho^{b\to sg}(p,q) = -\frac{1}{3}e\Big[\Sigma(p+q)-\Sigma(p)\Big]\;. \tag{24}
$$

To renormalize the $\bar s b\gamma$ vertex, we demand that the Ward-Takahashi identity be preserved for the renormalized vertex $\hat\Gamma_\rho^{b\to s\gamma}$[18],

$$
q^\rho\hat\Gamma_\rho^{b\to sg}(p,q) = -\frac{1}{3}e\Big[\hat\Sigma(p+q)-\hat\Sigma(p)\Big]\;. \tag{25}
$$

The other steps are similar to those applied in the calculation for the $\bar s b g$ vertex. The result is written as

$$
\mathcal{L}_{b\to s\gamma} = \frac{4G_F}{\sqrt2}V_{tb}V_{ts}^*\Big\{C_1(\mu_{\rm W})\mathcal{O}_1+C_4(\mu_{\rm W})\mathcal{O}_4+C_6(\mu_{\rm W})\mathcal{O}_6+C_7(\mu_{\rm W})\mathcal{O}_7+C_8(\mu_{\rm W})\mathcal{O}_8\Big\} \tag{26}
$$

with

$$
\begin{aligned}
C_6(\mu_{\rm W}) =& x_t\Big[\frac{18x_t^2-11x_t-1}{8(1-x_t)^3}+\frac{15x_t^2-16x_t+4}{4(1-x_t)^4}\ln x_t\Big]+\frac{x_t}{\tan^2\beta}\Big[\frac{4x_t^2+x_tx_{_H}+25x_{_H}^2}{72(x_t-x_{_H})^3} \\
&-\frac{(3x_t^2x_{_H}+2x_tx_{_H}^2)(\ln x_t-\ln x_{_H})}{12(x_t-x_{_H})^4}\Big]+\sum_{\alpha,\,\beta}(\mathcal{A}_3^{\alpha,\beta})^2\Big[-\frac{8x_{\tilde U_\alpha^3}^2+5x_{\tilde U_\alpha^3}x_{\chi_\beta}-7x_{\chi_\beta}^2}{36(x_{\tilde U_\alpha^3}-x_{\chi_\beta})^2}
\end{aligned}
$$

$$
+\frac{(3x^2_{\tilde{U}^3_\alpha}x_{\chi_\beta}-2x_{\tilde{U}^3_\alpha}x^2_{\chi_\beta})(\ln x_{\tilde{U}^3_\alpha}-\ln x_{\chi_\beta})}{6(x_{\tilde{U}^3_\alpha}-x_{\chi_\beta})^4}\Big],
$$

$$
\begin{aligned}
C_7(\mu_{\rm W}) = & \Big[\frac{-19x_t^3+25x_t^2}{12(1-x_t)^3}+\frac{3x_t^4-30x_t^3+54x_t^2-32x_t+8}{6(1-x_t)^4}\ln x_t\Big] \\
& +\frac{x_t}{\tan^2\beta}\Big[-\frac{19x_t^2+109x_tx_H-98x_H^2}{108(x_t-x_H)^3}+\frac{(3x_t^3-9x_t^2x_H-4x_H^3)(\ln x_t-\ln x_H)}{36(x_t-x_H)^4}\Big] \\
& +\sum_{\alpha,\,\beta}(\mathcal{A}_3^{\alpha,\beta})^2\Big[\frac{52x^2_{\tilde{U}^3_\alpha}-101x_{\tilde{U}^3_\alpha}x_{\chi_\beta}+43x^2_{\chi_\beta}}{54(x_{\tilde{U}^3_\alpha}-x_{\chi_\beta})^3} \\
& -\frac{(6x^3_{\tilde{U}^3_\alpha}-27x^2_{\tilde{U}^3_\alpha}x_{\chi_\beta}+12x_{\tilde{U}^3_\alpha}x^2_{\chi_\beta}+2x^3_{\chi_\beta})(\ln x_{\tilde{U}^3_\alpha}-\ln x_{\chi_\beta})}{9(x_{\tilde{U}^3_\alpha}-x_{\chi_\beta})^4}\Big], \\
C_8(\mu_{\rm W}) = & \, x_t\Big[\frac{-5x_t^2+8x_t-3}{4(1-x_t)^3}+\frac{3x_t-2}{2(1-x_t)^3}\ln x_t\Big] \\
& -x_t\Big[\frac{1}{2(x_t-x_H)}+\frac{(x_H^2+x_tx_H)(\ln x_t-\ln x_H)}{2(x_t-x_H)^3}\Big] \\
& +\sum_{\alpha,\,\beta}\frac{m_{\chi_\beta}}{\sqrt{2}m_{\rm W}\cos\beta}(\mathcal{A}_3^{\alpha,\beta}\mathcal{B}_3^{\alpha,\beta})\Big[-\frac{7x_{\tilde{U}^3_\alpha}-5x_{\chi_\beta}}{6(x_{\tilde{U}^3_\alpha}-x_{\chi_\beta})^2} \\
& +\frac{(3x^2_{\tilde{U}^3_\alpha}+2x_{\tilde{U}^3_\alpha}x_{\chi_\beta})(\ln x_{\tilde{U}^3_\alpha}-\ln x_{\chi_\beta})}{3(x_{\tilde{U}^3_\alpha}-x_{\chi_\beta})^3}\Big].
\end{aligned}
\tag{27}
$$

3 Numerical results

In this section, we present our numerical results of the Wilson coefficients in the mSUGRA model. As we have mentioned before, the model is fully specified by five parameters

$$
m_0,\ m_{\frac{1}{2}},\ A_0,\ \tan\beta,\ sgn(\mu).
$$

Here $m_0,\ m_{\frac{1}{2}},$ and A_0 are the universal scalar quark mass, gaugino mass and trilinear scalar coupling. They are assumed to arise through supersymmetry breaking in a hidden-sector at the GUT scale $\mu_{GUT}\simeq 2\times 10^{16}$GeV. In our numerical calculation, to maintain consistency of the theory and the up-to-date experimental observation, when we obtain the numerical value of the Higgs mass in the mSUGRA model with the five parameters, and we include all one-loop effects in the Higgs potential[19]. Moreover we also employ the two-loop RGEs[20] with one-loop threshold corrections[19, 21] as the energy scale runs down from the mSUGRA scale to the lower weak scale.

For the SM parameters, we have $m_b = 5$GeV, $m_t = 174$GeV, $m_{\rm W} = 80.23$GeV, $\alpha_e(m_{\rm W}) = \frac{1}{128}$, $\alpha_s(m_{\rm W}) = 0.12$ at the weak scale. In our later calculations, we always set $A_0 = 0$, $sgn(\mu) = -$. Taking above values, we find that the Standard Model prediction for the Wilson Coefficients are $C_{2_{SM}}(m_{\rm W}) = 0.315$, $C_{3_{SM}}(m_{\rm W}) = 0.256$, $C_{5_{SM}}(m_{\rm W}) = -0.218$, $C_{6_{SM}}(m_{\rm W}) = -1.477$, $C_{7_{SM}}(m_{\rm W}) = 1.511$, $C_{8_{SM}}(m_{\rm W}) = 0.891$. Then with the aforementioned inputs of the five parameter we evaluate the supersymmetric corrections to those Wilson coefficients $C_i(m_{\rm W})$ $(i = 2,\ 3,\ 5,\ \cdots, 8)$.

Even though other Wilson coefficients also get nonzero contributions from the supersymmetric sector, our discussions mainly focus on $C_{2,3,5,6,7,8}(m_{\rm W})$ because they play more significant roles in low energy phenomenol-

ogy. Moreover, we will illustrate their dependence on the supersymmetric parameters through the attached figures.

In Fig.4, we plot $C_2(m_{\rm W}),\ C_3(m_{\rm W}),\ C_5(m_{\rm W})$ versus $m_{\frac{1}{2}}$ with $m_0 = 100{\rm GeV},\ A_0 = 0,\ sgn(\mu) = -$ and $\tan\beta = 2,\ 20$. Setting $m_{\frac{1}{2}} = 100{\rm GeV},\ A_0 = 0,\ sgn(\mu) = -$ and $\tan\beta = 2,\ 20$, the dependence of $C_2(m_{\rm W}),\ C_3(m_{\rm W}),\ C_5(m_{\rm W})$ on m_0 is plotted in Fig.5. From Fig.4, we find that the supersymmetric contributions make the Wilson coefficients at the weak scale deviate from the SM predictions obviously when $m_{\frac{1}{2}} \leq 800{\rm GeV}$; when $m_{\frac{1}{2}}$ further increases, the new physics contributions gradually become immaterial. A similar situation exists in Fig.5, the Wilson coefficients tend to the SM prediction values as m_0 increases. For the Wilson coefficients of $b \to s\gamma$, we plot $C_6(m_{\rm W}),\ C_7(m_{\rm W}),\ C_8(m_{\rm W})$ versus $m_{\frac{1}{2}}$ with $m_0 = 100{\rm GeV},\ A_0 = 0,\ sgn(\mu) = -$ and $\tan\beta = 2,\ 20$ in Fig.6. Setting $m_{\frac{1}{2}} = 100{\rm GeV},\ A_0 = 0,\ sgn(\mu) = -$ and $\tan\beta = 2,\ 20$, dependence of $C_2(m_{\rm W}),\ C_3(m_{\rm W}),\ C_5(m_{\rm W})$ on m_0 is plotted in Fig.7. The trend of changes of those coefficients with respect to parameters $m_{\frac{1}{2}},\ m_0$ is similar to that in the $b \to sg$ case.

When the effective Lagrangian is applying at the hadronic scale, we should evolve those Wilson coefficients from the weak scale down to the hadronic scale. The running depends on the anomalous dimension matrix of concerned operators[22]. The coefficients $C_i(m_{\rm W})$ obtained at the weak scale M_W are regarded as the initial conditions for the differential RGEs.

4 Discussions

In this work, we discuss contributions to the effective Lagrangian for $b \to sg$ and $b \to s\gamma$ from the SUSY sector in the mSUGRA model. As many authors suggested, if the masses of the lightest SUSY particles are close to the electroweak energy scale, the contribution from the SUSY sector to the Wilson coefficients of the induced operators is comparable with that from SM.

Our numerical results indicate that within a reasonable mSUGRA parameter range, the SUSY contribution to $C_5(m_{\rm W})$ can enlarge the SM prediction by about 90%, and to the other coefficients, the SUSY contributions are not smaller than 30% of that of SM.

If the masses of the SUSY particles are larger, the SUSY contributions to the effective Lagrangian would become weaker. Then as the SUSY particles are very heavy, the main contribution to the effective Lagrangian uniquely comes from the standard model.

As well known, the QCD correction to the vertices $\bar{s}bg$ and $\bar{s}b\gamma$ is important and for practical application of the effective Lagrangian to evaluate the physical processes, say B decays, one needs to run down the coefficients from the weak energy scale to the hadronic scale, i.e. $M_B \sim 5{\rm GeV} \ll \mu_W$, in terms of the RGEs[4].

In this work, we adopt the non-linear R_ξ gauge. The advantage is that the Ward-Takahashi identity holds at the one-loop level no matter for the unrenormalized or renormalized quantities. This advantage would be more obvious as we go on doing the two-loop calculations.

Our numerical results also show that as all SUSY particles become very heavy, the values of all coefficients tend to that determined by the SM sector which is consistent with the results obtained before [18].

Acknowledgements

This work is partially supported by the National Natural Science Foundation of China.

Appendix

A The expressions for the form factors

The form factors in self-energy is given as

$$
\begin{aligned}
A_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) &= (1+\frac{x_i}{2})\Big[\Delta+\frac{1}{2}+\ln x_\mu+\frac{x_i}{x_i-1}-\frac{x_i^2\ln x_i}{(x_i-1)^2}\Big] \\
&+\frac{x_i}{2\tan^2\beta}\Big[\Delta+\frac{1}{2}+\ln x_\mu+\frac{x_i}{x_i-x_H}-\frac{x_i^2\ln x_i}{(x_i-x_H)^2}+\frac{(2x_Hx_i-x_H^2)\ln x_H}{(x_i-x_H)^2}\Big] \\
&+\sum_{\alpha,\,\beta}(\mathcal{A}_i^{\alpha,\beta})^2\Big[\Delta+\frac{3}{2}+\ln x_\mu-\frac{x_{\tilde{U}^i_\alpha}}{x_{\tilde{U}^i_\alpha}-x_{\chi_\beta}}+\frac{x_{\tilde{U}^i_\alpha}(2x_{\chi_\beta}-x_{\tilde{U}^i_\alpha})\ln x_{\tilde{U}^i_\alpha}}{(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^2}-\frac{x_{\chi_\beta}^2\ln x_{\chi_\beta}}{(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^2}\Big]\;, \\
A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) &= (1+\frac{x_i}{2})\Big[\frac{2x_i^2+5x_i-1}{3(x_i-1)^3}-\frac{2x_i^2\ln x_i}{(x_i-1)^4}\Big] \\
&+\frac{x_i}{2\tan^2\beta}\Big[\frac{2x_i^2+5x_ix_H-x_H^2}{3(x_i-x_H)^3}-\frac{2x_i^2x_H(\ln x_i-\ln x_H)}{(x_i-x_H)^4}\Big] \\
&+\sum_{\alpha,\,\beta}(\mathcal{A}_i^{\alpha,\beta})^2\Big[\frac{x_{\tilde{U}^i_\alpha}^2-5x_{\tilde{U}^i_\alpha}x_{\chi_\beta}-2x_{\chi_\beta}^2}{3(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^3}+\frac{2x_{\tilde{U}^i_\alpha}x_{\chi_\beta}(\ln x_{\tilde{U}^i_\alpha}-\ln x_{\chi_\beta})}{(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^4}\Big]\;, \\
B_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) &= \frac{x_i^2\ln x_i}{x_i-1}-\frac{x_i^2\ln x_i-x_ix_H\ln x_H}{x_i-x_H} \\
&+2\sum_{\alpha,\,\beta}\frac{m_{\chi_\beta}}{\sqrt{2}m_{\rm W}\cos\beta}(\mathcal{A}_3^{\alpha,\beta}\mathcal{B}_3^{\alpha,\beta})\Big[\Delta+1+\ln x_\mu-\frac{x_{\tilde{U}^i_\alpha}\ln x_{\tilde{U}^i_\alpha}-x_{\chi_\beta}\ln x_{\chi_\beta}}{x_{\tilde{U}^i_\alpha}-x_{\chi_\beta}}\Big]\;, \\
B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) &= -x_i\Big[\frac{x_i+1}{2(x_i-1)^2}-\frac{x_i\ln x_i}{(x_i-1)^3}\Big] \\
&+x_i\Big[\frac{x_i+x_H}{2(x_i-x_H)^2}-\frac{x_ix_H(\ln x_i-\ln x_H)}{(x_i-x_H)^3}\Big] \\
&+\sum_{\alpha,\,\beta}\frac{m_{\chi_\beta}}{\sqrt{2}m_{\rm W}\cos\beta}(\mathcal{A}_3^{\alpha,\beta}\mathcal{B}_3^{\alpha,\beta})\Big[\frac{x_{\tilde{U}^i_\alpha}+x_{\chi_\beta}}{(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^2}-\frac{2x_{\tilde{U}^i_\alpha}x_{\chi_\beta}(\ln x_{\tilde{U}^i_\alpha}-\ln x_{\chi_\beta})}{(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^3}\Big]\;, \\
C_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) &= \frac{1}{2}\Big[\Delta+\frac{1}{2}+\ln x_\mu+\frac{x_i}{x_i-1}-\frac{x_i^2\ln x_i}{(x_i-1)^2}\Big] \\
&+\frac{\tan^2\beta}{2}\Big[\Delta+\frac{1}{2}+\ln x_\mu+\frac{x_i}{x_i-x_H}-\frac{x_i^2\ln x_i}{(x_i-x_H)^2}+\frac{(2x_Hx_i-x_H^2)\ln x_H}{(x_i-x_H)^2}\Big] \\
&+\sum_{\alpha,\,\beta}\frac{(\mathcal{B}_3^{\alpha,\beta})^2}{2\cos^2\beta}\Big[\Delta+\frac{3}{2}+\ln x_\mu-\frac{x_{\tilde{U}^i_\alpha}}{x_{\tilde{U}^i_\alpha}-x_{\chi_\beta}}+\frac{x_{\tilde{U}^i_\alpha}(2x_{\chi_\beta}-x_{\tilde{U}^i_\alpha})\ln x_{\tilde{U}^i_\alpha}}{(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^2} \\
&-\frac{x_{\chi_\beta}^2\ln x_{\chi_\beta}}{(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^2}\Big]\;.
\end{aligned}
\tag{28}
$$

The expression for $F_i(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})$ $(i=1,\;\cdots,\;5)$ is written as

$$
F_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}) = (1+\frac{x_i}{2})\Big[\frac{5x_i^2-22x_i+5}{18(x_i-1)^3}+\frac{(3x_i-1)\ln x_i}{3(x_i-1)^4}\Big]
$$

$$+\frac{1}{\tan^2\beta}\Big[\frac{x_i(5x_i^2-22x_ix_{_H}+5x_{_H}^2)}{36(x_i-x_{_H})^3}-\frac{(x_{_H}^3x_i-3x_{_H}^2x_i^2)(\ln x_i-\ln x_{_H})}{6(x_i-x_{_H})^4}\Big]$$

$$+\sum_{\alpha,\,\beta}(\mathcal{A}_i^{\alpha,\beta})^2\Big[\frac{x_{\tilde{U}_\alpha^i}^2-8x_{\tilde{U}_\alpha^i}x_{\chi_\beta}-17x_{\chi_\beta}^2}{36(x_{\tilde{U}_\alpha^i}-x_{\chi_\beta})^3}+\frac{(3x_{\tilde{U}_\alpha^i}x_{\chi_\beta}^2+x_{\chi_\beta}^3)(\ln x_{\tilde{U}_\alpha^i}-\ln x_{\chi_\beta})}{6(x_{\tilde{U}_\alpha^i}-x_{\chi_\beta})^4}\Big]\,,$$

$$F_2(x_i,x_{_H},x_{\tilde{U}_\alpha^i},x_{\chi_\beta})=\Big[-\frac{x_i^3-15x_i^2-12x_i+8}{12(x_i-1)^3}-\frac{(5x_i^2-2x_i)\ln x_i}{2(x_i-1)^4}\Big]$$

$$+\frac{1}{\tan^2\beta}\Big[-\frac{x_i(x_i^2-5x_ix_{_H}-2x_{_H}^2)}{12(x_i-x_{_H})^3}-\frac{x_i^2x_{_H}^2(\ln x_i-\ln x_{_H})}{2(x_i-x_{_H})^4}\Big]$$

$$+\sum_{\alpha,\,\beta}(\mathcal{A}_i^{\alpha,\beta})^2\Big[-\frac{x_{\tilde{U}_\alpha^i}^2-x_{\tilde{U}_\alpha^i}x_{\chi_\beta}-2x_{\chi_\beta}^2}{6(x_{\tilde{U}_\alpha^i}-x_{\chi_\beta})^3}-\frac{x_{\tilde{U}_\alpha^i}x_{\chi_\beta}^2(\ln x_{\tilde{U}_\alpha^i}-\ln x_{\chi_\beta})}{(x_{\tilde{U}_\alpha^i}-x_{\chi_\beta})^4}\Big]\,,$$

$$F_3(x_i,x_{_H},x_{\tilde{U}_\alpha^i},x_{\chi_\beta})=\Big[\frac{5x_i^3+3x_i^2+6x_i+4}{12(x_i-1)^3}+\frac{x_i(2x_i^2-x_i-2)\ln x_i}{2(x_i-1)^4}\Big]$$

$$+\frac{1}{\tan^2\beta}\Big[\frac{x_i(5x_i^2+5x_ix_{_H}-4x_{_H}^2)}{12(x_i-x_{_H})^3}+\frac{(2x_i^3x_{_H}-x_i^2x_{_H}^2)(\ln x_i-\ln x_{_H})}{2(x_i-x_{_H})^4}\Big]$$

$$+\sum_{\alpha,\,\beta}(\mathcal{A}_i^{\alpha,\beta})^2\Big[-\frac{x_{\tilde{U}_\alpha^i}^2-x_{\tilde{U}_\alpha^i}x_{\chi_\beta}-2x_{\chi_\beta}^2}{6(x_{\tilde{U}_\alpha^i}-x_{\chi_\beta})^3}-\frac{x_{\tilde{U}_\alpha^i}x_{\chi_\beta}^2(\ln x_{\tilde{U}_\alpha^i}-\ln x_{\chi_\beta})}{(x_{\tilde{U}_\alpha^i}-x_{\chi_\beta})^4}\Big]\,,$$

$$F_4(x_i,x_{_H},x_{\tilde{U}_\alpha^i},x_{\chi_\beta})=-(1+\frac{x_i}{2})\Big[\frac{5x_i^2-22x_i+5}{18(x_i-1)^3}+\frac{(3x_i-1)\ln x_i}{3(x_i-1)^4}\Big]$$

$$+\frac{1}{\tan^2\beta}\Big[-\frac{x_i(5x_i^2-22x_ix_{_H}+5x_{_H}^2)}{36(x_i-x_{_H})^3}+\frac{(x_{_H}^3x_i-3x_{_H}^2x_i^2)(\ln x_i-\ln x_{_H})}{6(x_i-x_{_H})^4}\Big]$$

$$+\sum_{\alpha,\,\beta}(\mathcal{A}_i^{\alpha,\beta})^2\Big[\frac{x_{\tilde{U}_\alpha^i}^2-8x_{\tilde{U}_\alpha^i}x_{\chi_\beta}-17x_{\chi_\beta}^2}{36(x_{\tilde{U}_\alpha^i}-x_{\chi_\beta})^3}+\frac{(3x_{\tilde{U}_\alpha^i}x_{\chi_\beta}^2+x_{\chi_\beta}^3)(\ln x_{\tilde{U}_\alpha^i}-\ln x_{\chi_\beta})}{6(x_{\tilde{U}_\alpha^i}-x_{\chi_\beta})^4}\Big]\,,$$

$$F_5(x_i,x_{_H},x_{\tilde{U}_\alpha^i},x_{\chi_\beta})=\Big[-\frac{x_i(x_i-3)}{4(x_i-1)^2}-\frac{x_i\ln x_i}{2(x_i-1)^3}\Big]+\Big[\frac{x_i(x_i-3x_{_H})}{4(x_i-x_{_H})^2}+\frac{x_ix_{_H}^2(\ln x_i-\ln x_{_H})}{2(x_i-x_{_H})^3}\Big]$$

$$+\sum_{\alpha,\,\beta}\frac{m_{\chi_\beta}}{\sqrt{2}m_{\mathrm{W}}\cos\beta}(\mathcal{A}_3^{\alpha,\beta}\mathcal{B}_3^{\alpha,\beta})\Big[\frac{x_{\tilde{U}_\alpha^i}+x_{\chi_\beta}}{2(x_{\tilde{U}_\alpha^i}-x_{\chi_\beta})^2}+\frac{x_{\tilde{U}_\alpha^i}x_{\chi_\beta}(-\ln x_{\tilde{U}_\alpha^i}+\ln x_{\chi_\beta})}{(x_{\tilde{U}_\alpha^i}-x_{\chi_\beta})^3}\Big]\,. \qquad (29)$$

References

[1] E. Gabrielli *et al.* Phys. Lett. **B**374(1996)80.

[2] M. Ciuchini, G. Degrassi, P. Gambino and G. F. Giudice, Nucl. Phys. **B527**(1998)21 (hep-ph/9710335); P. Ciafaloni, A. Romanino and A. Strumia, Nucl. Phys. **B524**(1998)361 (hep-ph/9710312); F. Borzumati and C. Greub, Phys. Rev. **D58**(1998)074004 (hep-ph/9802391); S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, Nucl. Phys. **B**353(1991)591; R. Barbieri and G. F. Giudice, Phys. Lett. **B**309(1993)86; F. Borzumati and C. Greub, T. Hurth and D. Wyler, Phys. Rev. **D62**(2000)075005 (hep-ph/9911245); M. Ciuchini, G. Degrassi, P. Gambino and G. F. Giudice, Nucl. Phys. **B534**(1998)3 (hep-ph/9806308); S. Bertolini and J. Matias, Phys. Rev. **D57**(1998)4197 (hep-ph/9709330); W. N. Cottingham, H. Mehrban and I. B. Whittingham, Phys. Rev. **D60**(1999)114029 (hep-ph/9905300); G. Barenboim and M. Raidal,

Phys. Lett. **B457**(1999)109 (hep-ph/9903270); J. L. Hewett and D. Wells, Phys. Rev. **D**55(1997)5549; A. Ali, P. Ball, L. T. Handoko and G. Hiller, Phys. Rev. **D61**(2000)074024 (hep-ph/9910221); A.Masiero and L. Silvestrini, hep-ph/9709244; hep-ph/9711401; M. Ciuchini et.al, JHEP 9810(1998)008; R. Contion, I. Scimemi, Eur. Phys. J. **C**10(1999)347; F. Krauss, G. Soff, hep-ph/9807238; Tai-Fu Feng, Xue-Qian Li, Wen-Gan Ma and Feng Zhang, Phys. Rev. **D63**(2001)015013; C. Greub and P. Liniger, Phys. Lett. **B494**(2000)237 (hep-ph/0008071), Phys. Rev. **D63**(2001)054025 (hep-ph/0009144); C. Greub, T. Hurth and D. Wyler, Phys. Rev. **D54**(1996)3350; A. Buras, M. Misiak and J. Urban, Nucl. Phys. **B**586(2000)397; M. Causse and J. Orloff, hep-ph/0012113; S. Prelovsek and D. Wyler, hep-ph/0012116.

[3] J. Feng, K. Matchev and F. Wilczek, astro-ph/0008115.

[4] Tai-Fu Feng, Xue-Qian Li and Guo-Li Wang, hep-ph/0101081.

[5] M. Hirsch, M. A. Diaz, W. Porod, J. C. Romao and J. W. F. Valle, Phys. Rev. **D62**(2000)113008.

[6] S. Weinberg, Phys. Lett. **B91**(1980)51.

[7] N. G. Deshpande and G. Eilam, Phys. Rev. **D26**(1982)2463.

[8] N. G. Deshpande and M. Nazerimonfared, Nucl. Phys. **B213**(1983)390.

[9] E. P. Shabalin, Sov. J. Nucl. Phys. **28**(1978)75.

[10] D. V. Nanopoulos, A. Yildiz and P. H. Cox, Ann. Phys. **127**(1980)126.

[11] S. P. Chia and S. Nandi, Phys. Lett. **B117**(1982)245.

[12] T. Inami and C. S. Lim, Prog. Theor. Phys. **65**(1981)297.

[13] R. Grigjanis, P. O'Donnell, M. Sutherland and H. Navelet, Phys. Rep. **228**(1993)93; Phys. Lett. **B213**(1988)355; **B286**(1992)413(E).

[14] G. Buchalla, A. J. Buras and M. Lautenbacher, Rev. Mod. Phys. **68**(1996)1125.

[15] B. Grinstein, R. Springer and M. Wise, Phys. Lett. **B202**(1988)138; Nucl. Phys. **B339**(1990)269.

[16] G. Cella, G. Curci, G. Ricciardi and A. Vicere, Phys. Lett. **B248**(1990)181.

[17] M. Misiak, Phys. Lett. **B269**(1991)161.

[18] S. P. Chia, Phys. Lett. **B130**(1983)315.

[19] D. Pierce, J. Bagger, K. Matchev and R. Zhang, Nucl. Phys. **B491**(1997)3.

[20] S. Martin, M. Vaughn, Phys. Rev. **D50**(1994)2282; I. Jack and D. Jones, Phys. Lett. **B333**(1994)372; Y. Yamada, Phys. Rev. **D50**(1994)3537; I. Jack, D. Jones, S. Martin, M. Vaughn and Y. Yamada, Phys. Rev. **D50**(1994)5481.

[21] J. Bagger, K. Matchev and D. Pierce, Phys. Lett. **B348**(1995)443.

[22] A. Buras, hep-ph/0101336.

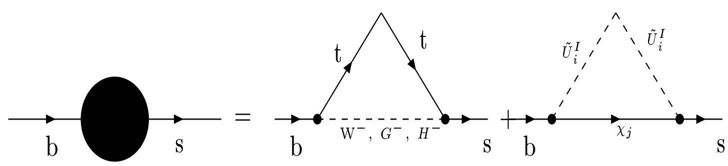


Figure 1: The one-loop self-energy diagrams for $b \to s$ in the SUSY model with minimal flavor violation

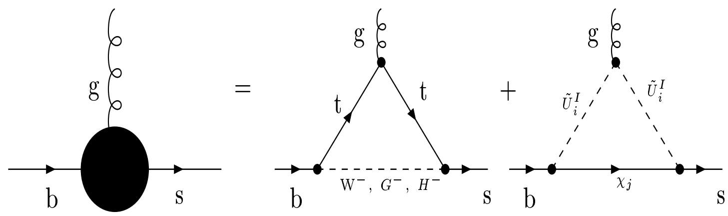


Figure 2: The one-loop diagrams for $b \to sg$ in the SUSY model with minimal flavor violation

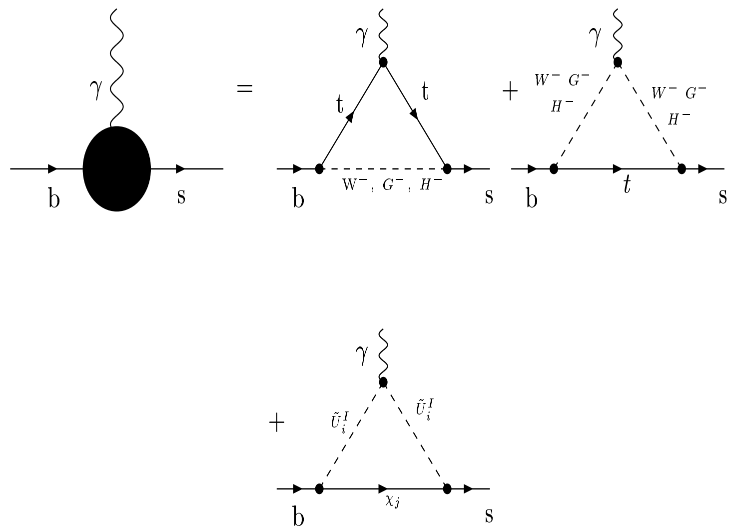
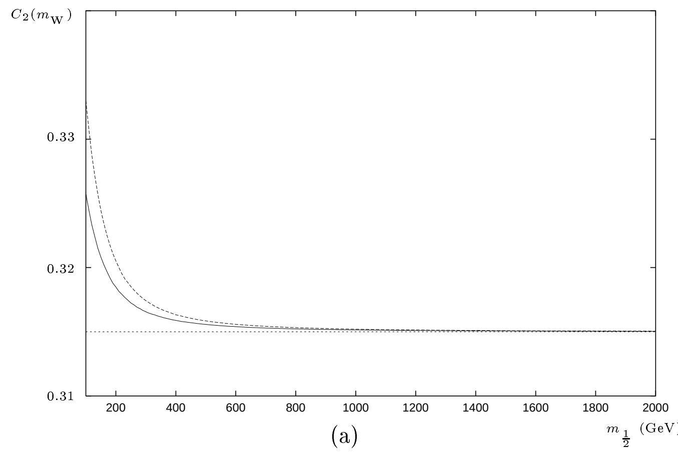
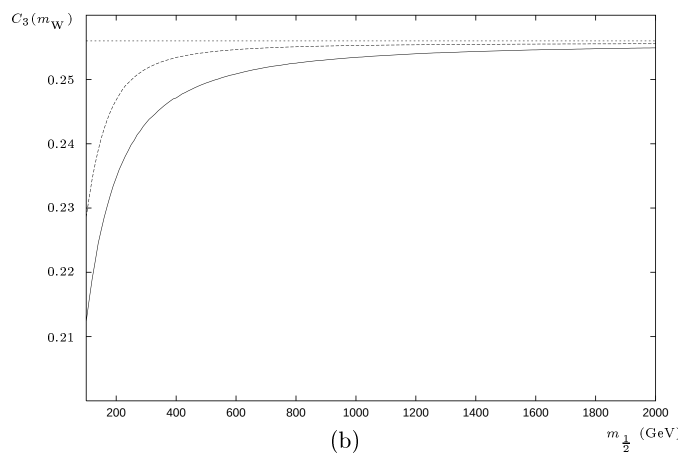


Figure 3: The one-loop diagrams for $b \to s\gamma$ in the SUSY model with minimal flavor violation



(a)



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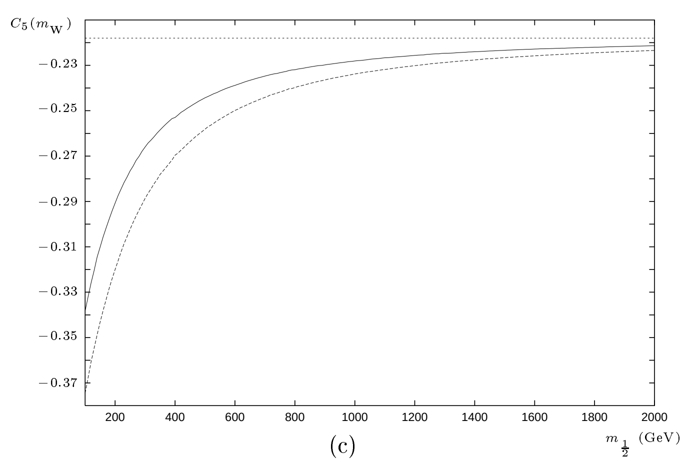
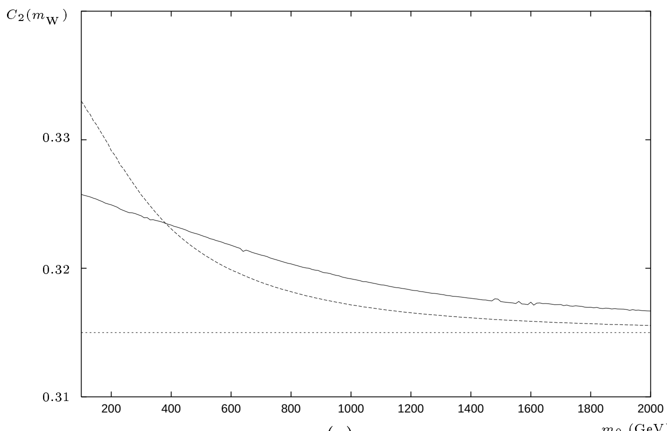
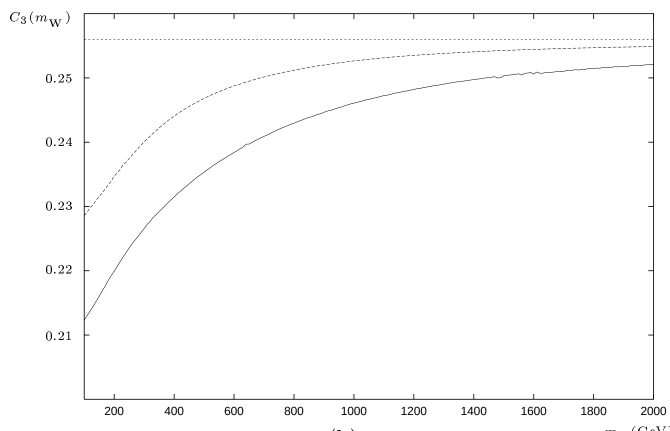


(c)

Figure 4: The Wilson coefficients at the weak scale for $b \to sg$ in mSUGRA versus $m_{\frac{1}{2}}$ with $A_0 = 0$, $sgn(\mu) = -$ and (i) solid-line: $\tan\beta = 2$; (ii) dash-line: $\tan\beta = 20$; (iii) dot-line: the SM prediction value.



(a)



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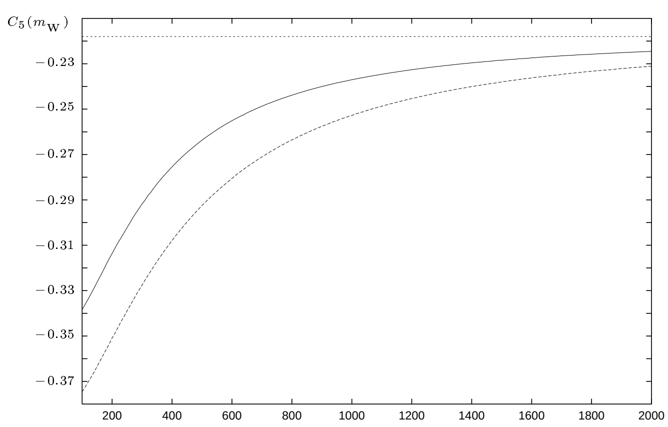

(c)

Figure 5: The Wilson coefficients at the weak scale for $b \to sg$ in mSUGRA versus m_0^2 with $A_0 = 0,\ sgn(\mu) = -$ and (i) solid-line: $\tan\beta = 2$; (ii) dash-line: $\tan\beta = 20$; (iii) dot-line: the SM prediction value.

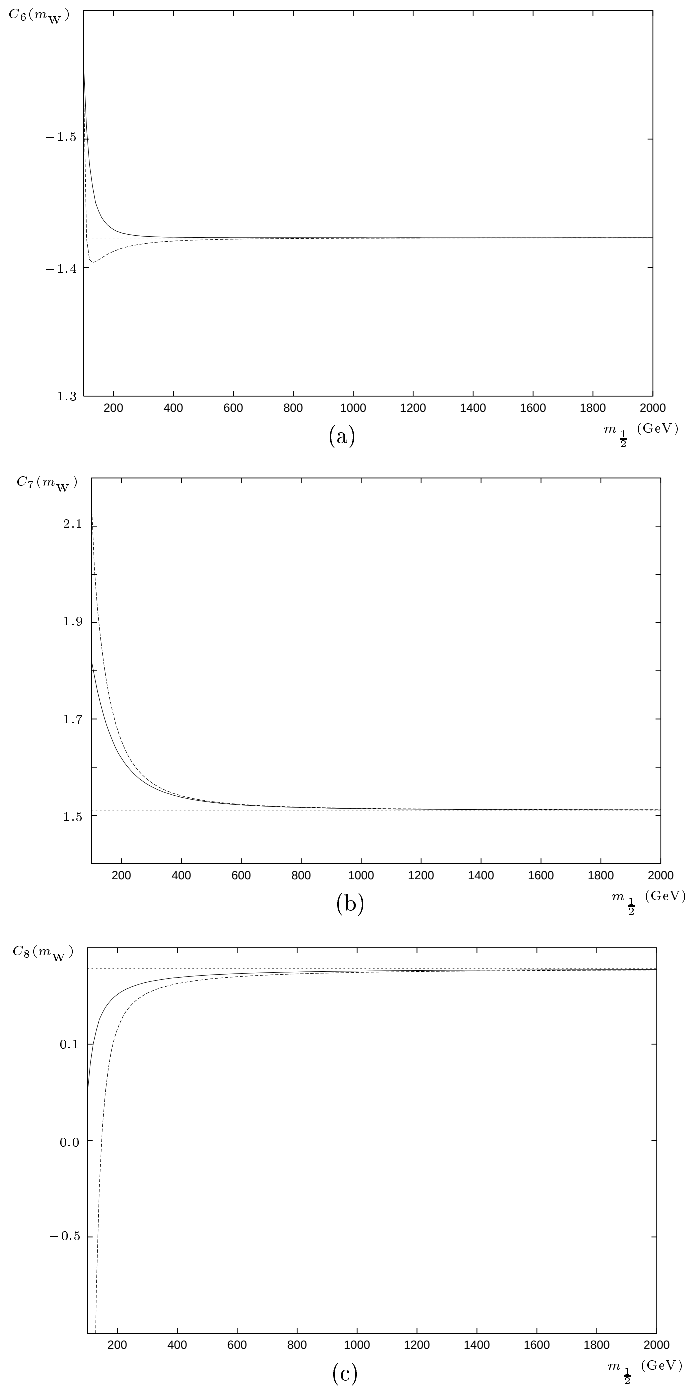

Figure 6: The Wilson coefficients at the weak scale for $b \to s\gamma$ in mSUGRA versus $m_{\frac{1}{2}}$ with $A_0 = 0,\ sgn(\mu) = -$ and (i) solid-line: $\tan\beta = 2$; (ii) dash-line: $\tan\beta = 20$; (iii) dot-line: the SM prediction value.

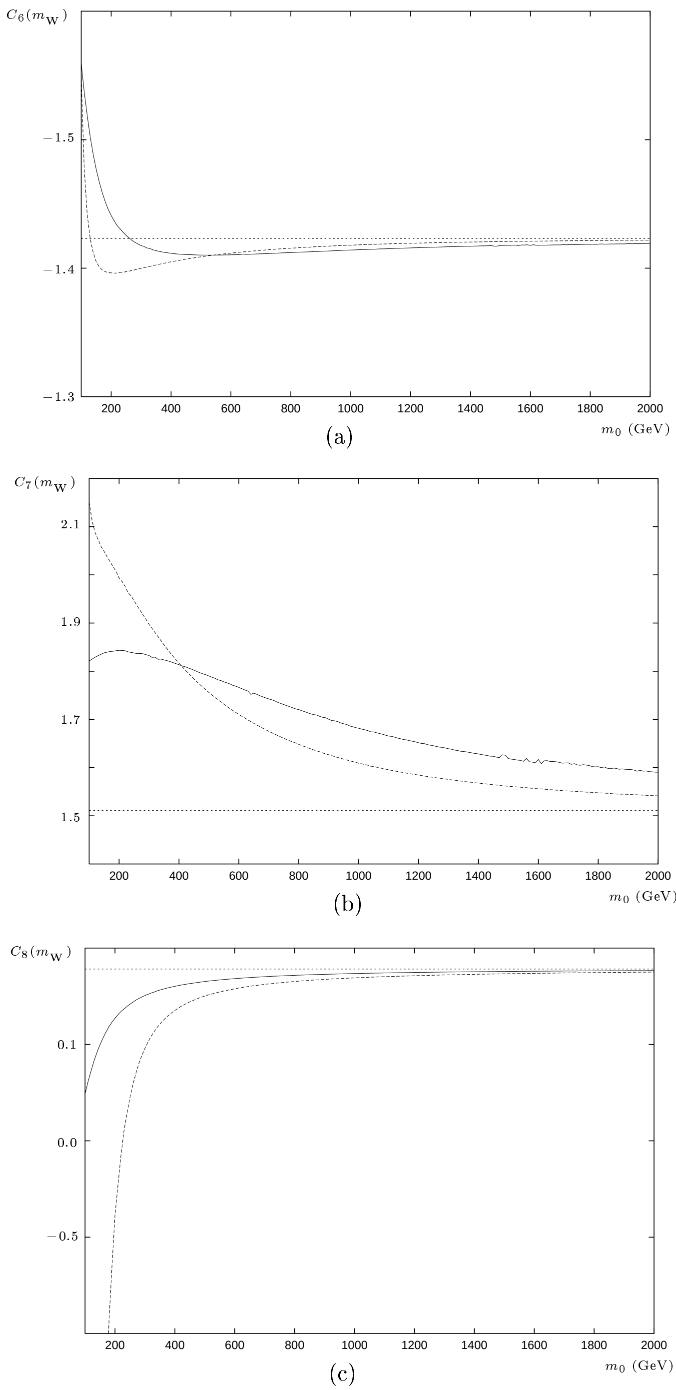

Figure 7: The Wilson coefficients at the weak scale for $b \to s\gamma$ in mSUGRA versus m_0^2 with $A_0 = 0,\ sgn(\mu) = -$ and (i) solid-line: $\tan\beta = 2$; (ii) dash-line: $\tan\beta = 20$; (iii) dot-line: the SM prediction value.